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Statistical self-similarity of hotspot seamount volumes modeled as self-similar criticality

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Abstract. The processes responsible for hotspot seamount formation are complex, yet the cumulative frequency-volume distribution of hotspot seamounts in the Easter Island/Salas y Gomez Chain (ESC) is found to be well-described by an upper-truncated power law. We develop a model for hotspot seamount formation where uniform energy input produces events initiated on a self-similar distribution of critical cells. We call this model Self-Similar Criticality (SSC). By allowing the spatial distribution of magma migration to be self-similar, the SSC model recreates the observed ESC seamount volume distribution. The SSC model may have broad applicability to other natural systems.

Introduction

The size and spatial distributions of seamounts (submarine volcanoes) have been used to discern the location, size and number of mantle plumes transporting heat to the base of the lithosphere [*Malamud and Turcotte*, 1999 and references therein]. *Scheirer and Macdonald* [1995] noted that these distributions may provide insight into locations of magma migration through the earth's crust.

Most previous work on seamount size distributions used height measurements [*Smith and Jordan*, 1988; *Wessel and Lyons*, 1997]. An exception was a study of North Pacific seamounts where the non-cumulative frequency-volume distribution was suggested to be "poisson-like" and no function was fit to the data [*Batiza*, 1982]. Volume is a more meaningful measure than height, as it provides a three-dimensional size measurement. We examine the cumulative frequency-volume distribution (CFVD) of 383 ESC seamounts over 200 m in height (including two volcanic islands) identified and measured by *Rappaport et al.* [1997] using SeaBeam and GLORI-B swath bathymetry maps (Figure 1). The base of each seamount was located at the sharp break in slope from the surrounding seafloor and each volume was determined by summing the volumes above each pixel in the basal area [*Rappaport*, 1996].

Analysis

A power law for a cumulative frequency-size distribution has the form $N(r) = Cr^{-\alpha}$, where $N(r)$ is the number of objects with characteristic size greater than or equal to object size r, $-\alpha$ is the slope on a log-log plot and C is a constant equal to the number of objects with size $r \geq 1$. The exponent α is related to the fractal dimension, D. When r is a linear, area, or volume measurement, D is equal to α, 2α, or 3α, respectively. An upper-truncated power law, $M(r)$, has the form

$$M(r) = C(r^{-\alpha} - r_T^{-\alpha}) \qquad (1)$$

where $M(r)$ is the number of objects with characteristic size greater than or equal to r, and there are no objects of size r_T or larger [*Burroughs and Tebbens*, in press]. Since each value in a cumulative distribution includes all larger objects, upper truncation of the distribution decreases the cumulative number associated with each object size. In equation (1), the second term, $Cr_T^{-\alpha}$, represents this decrease from the power law, $Cr^{-\alpha}$.

Applying Equation (1) to the ESC data, $M(r)$ is the observed cumulative number and r is seamount volume. The ESC seamount CFVD is well described by equation (1) with $C = 1270$, $\alpha = 0.57$, and $r_T = 5360\,\mathrm{km}^3$ (Figure 2). Because we examine seamount volume, the fractal dimension is 3α, thus D equals 1.71.

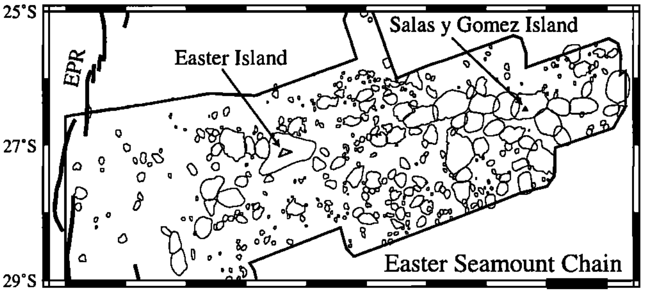

Figure 1. Easter Island/Salas y Gomez hotspot seamount chain showing the location and basal outlines for 383 seamounts measured by *Rappaport et al.* [1997]. EPR is the East Pacific Rise plate boundary.

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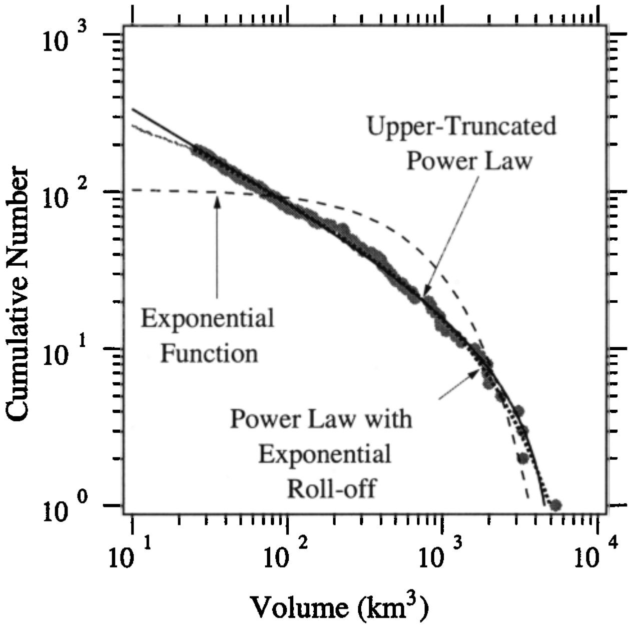


Figure 2. Cumulative frequency-volume distribution for the seamounts shown in Figure 1 fit with an upper-truncated power law (solid line), an exponential function (dashed line), and a power law with an exponential roll-off (dotted line).

Cumulative height distributions for seamounts along the ridge axis follow an exponential function [e.g., *Magde and Smith,* 1995]. We find an exponential function of the form $N(r) = N_0 e^{-ar}$ does not provide a good description of the ESC volume distribution (Figure 2).

A power law with an exponential roll-off has been used to describe cumulative distributions of earthquake magnitudes when the distribution falls away from a power law at large magnitude [*Kagan,* 1993; *Main,* 2000]. A power law with an exponential roll-off,

$$N(r) = Cr^{-\alpha}e^{-ar}, \tag{2}$$

is fit to the ESC seamount CFVD and is nearly indistinguishable from an upper-truncated power law (Figure 2).

There is no physical evidence to support equation (2), which requires both a power law and an exponential function to describe the seamount distribution. An upper-truncated power law describes the observed distribution without introducing an exponential term and has a reasonable physical explanation. Maximum volume may be limited by the movement of the lithospheric plate over the hotspot, restricting the amount of magma supplied to any given seamount.

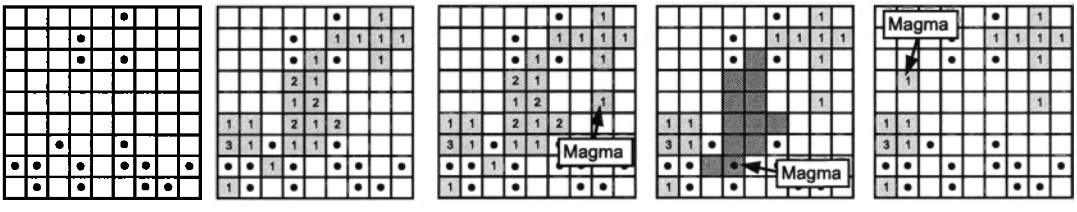
Self-Organized Criticality

Many natural phenomena that exhibit power-law frequency-size distributions are modeled as self-organized critical (SOC) systems [e.g., *Bak et al.,* 1987]. We attempted to apply an SOC model to seamount formation. The slider block, forest fire, and sand pile SOC models all produce non-cumulative frequency-size distributions with scaling exponents near unity [e.g., *Turcotte,* 1999], inconsistent with the ESC seamount CFVD where the scaling exponent, α, is 0.57. In an attempt to better replicate the observed distribution, we modified existing SOC models, but failed to reproduce the observed CFVD and scaling exponent.

Self-Similar Criticality

We develop a model of Self-Similar Criticality (SSC) that generates power law cumulative distributions with a range of scaling exponents. A square grid contains a stochastic fractal pattern of critical grid cells, the only fractal distribution assumed by the model. The model consists of adding units of material to randomly selected grid cells in successive steps (Figure 3). An event occurs when material is added to a critical cell, causing the critical cell and all occupied non-diagonal adjacent cells to empty. Grid cells selected as critical do not change and remain in the critical state at all times. Event size is the number of units of material emptied from the grid in each event. Applying the SSC model to hotspot seamount formation, each unit of material represents a volume of magma forming within the lithosphere. The critical cells represent locations of weakness in the lithosphere where magma can rise to form a seamount. An event is the formation of a seamount and event size is seamount volume. When magma is added to a critical cell, the newly added magma, together with all magma in non-diagonal adjacent cells, coalesces and forms a seamount. Although this model does not move the grid cells to simulate a moving lithospheric plate, each event in the model produces a separate volcano, equivalent to forming lines of volcanoes on a moving plate.

To create a stochastic fractal pattern [e.g., *Mandelbrot,* 1982, p. 219] of critical grid cells, we start with a square grid generator of size n_r rows by n_r columns. The generator contains a fixed number, n_c, of randomly located critical cells. The first order of this fractal is the generator. The second



Critical Cells Sample Configuration Step 1 Step 2, Eruption Step 3

Figure 3. Illustration of the SSC model. Critical cells (black dots) are a second order fractal with $n_r = 3$, $n_c = 4$ and $D_c = 1.26$. In the sample configuration, the model has been running and gray cells contain the labeled number of magma units. In Step 1 magma is added to a non-critical cell. In Step 2 magma is added to a critical cell and 16 units of magma leave the grid to form a seamount.

order stochastic fractal is obtained by replacing each critical cell of the first order fractal with the generator containing n_c critical cells at reselected random locations. The process is repeated to create higher order fractal patterns. The spatial fractal dimension of the critical cells, D_c, is $\ln n_c / \ln n_r$.

We create three third order stochastic fractal patterns with n_r equal to 5, producing grids of 125 by 125 cells. By selecting n_c to be 4, 6, and 8, we create stochastic fractals with D_c equal to 0.86, 1.11, and 1.29, respectively. Running the SSC model for 10^7 iterations on grids with these three patterns of critical cells, we obtain cumulative frequency-size distributions that follow equation (1) with scaling exponents, α, of 0.34, 0.57 and 0.70 (Figure 4). The scaling exponent of the resulting cumulative frequency-size distribution depends on the spatial fractal dimension of the critical cells. The model with D_c equal to 1.11 produces a scaling exponent of 0.57, equal to that observed for the seamount volumes of the ESC region. Fewer iterations of the SSC model produce a distribution with fewer data points and a less prominent roll-off (Figure 4), similar to the distribution for ESC seamount volumes.

The SSC model applied to seamount formation

Figure 5 illustrates a conceptual application of the SSC model to hotspot seamount formation. Heat is uniformly supplied to the base of the lithosphere. Following the model of *Pan and Batiza* [1998] for the ESC region, magma collects within the lithosphere at the base of the crust. The fractally-distributed critical cells represent locations of magma migration through the lithosphere. A fractal distribution is consistent with *Shaw and Chouet's* [1991] suggestion that spatial patterns of magma transport may be fractal over a range of scales from 0.1 mm up to 10^3 km.

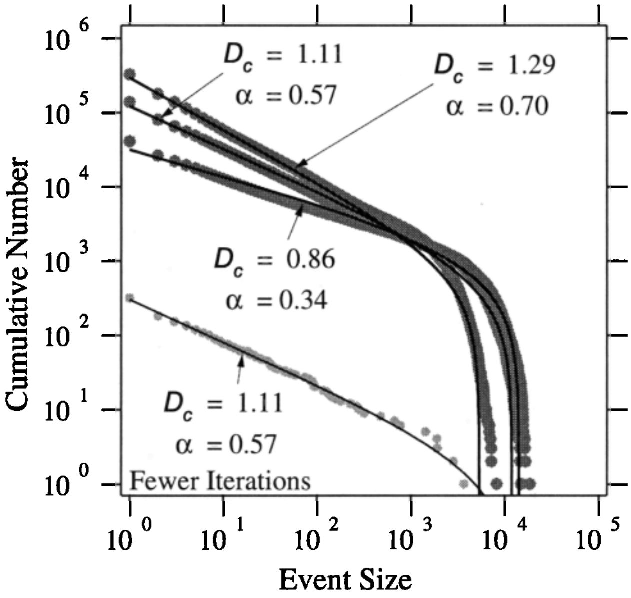


Figure 4. Cumulative frequency-volume distribution for the SSC model run for 1 x 10^7 iterations. Input values for D_c of 0.86, 1.11, and 1.29, result in cumulative frequency-size distributions that follow an upper-truncated power law with scaling exponents, α, of 0.34, 0.57 and 0.70, respectively. When $D_c = 1.11$, $\alpha = 0.57$, as observed for the ESC seamount CFVD. A model run for 2 x 10^4 iterations produces a lower activity level, similar to the ESC distribution in Figure 2.

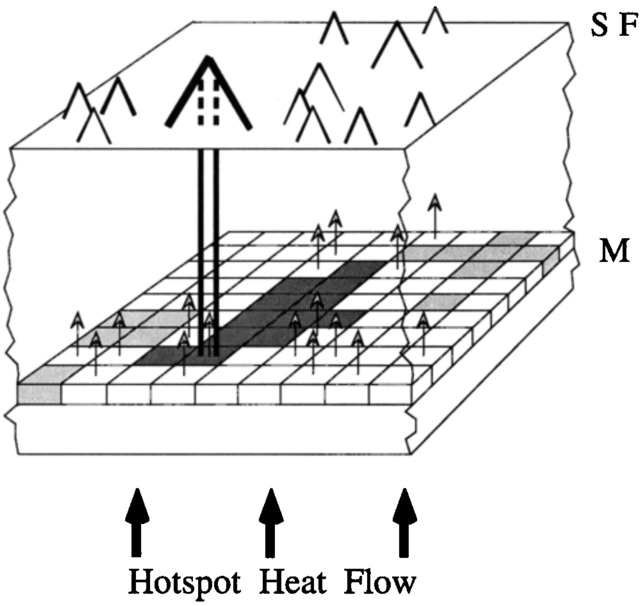


Figure 5. SSC model applied to hotspot seamount formation. Heavy arrows represent uniform heat input. Magma collects within the lithosphere at the base of the crust (M). Critical cells are shown with small vertical arrows. Cells containing magma are light gray. Cells forming the seamount on the seafloor (SF) in this step are dark gray. The diagram shows the configuration in step 2 of Figure 3. Crustal thickness is not to scale and conduit shape is not known.

The ability of the SSC model to replicate the observed ESC seamount CFVD suggests that locations of magma migration through the lithosphere may be restricted to a statistically self-similar distribution. To replicate the ESC seamount CFVD, the fractal dimension of critical cells in the SSC model is close to one ($D_c = 1.1$). In the ESC region, magma migration may occur in locations of weakness associated with tectonic features such as abyssal hill faults, fracture zones, pseudofaults, and/or intersections of these features. Several of these tectonic features have fractal properties with fractal dimensions between 1.2 and 1.5 [*Barton*, 1995; *Gilbert and Malinverno*, 1988; *Malinverno*, 1995]. Locations of magma migration may also be influenced by crustal thickness variations [*McNutt et al.*, 1989].

Factors not considered when applying the SSC model to ESC seamount formation are interactions between plume and ridge axis upwelling [*Pan and Batiza*, 1998], crustal thinning caused by the hot spot [*Kingsley and Schilling*, 1998], and tectonic processes associated with super-fast seafloor spreading [*Hey et al.*, 1985].

Continental basaltic eruptions exhibit self-similar spatial and temporal clustering over a wide range of scales and a model has been developed that replicates these results [*Pelletier*, 1999]. We make no conclusions regarding spatial and temporal clustering of ESC seamounts because the length scales span less than one order of magnitude and temporal resolution is too limited.

The SSC model may be applied to other natural systems with observed fractal characteristics. For example, fractal topography could be incorporated into existing models of avalanches or forest fires [e.g., *Bak et al.*, 1987, 1990].

Conclusions

1. The CFVD of seamounts in the ESC region follows an upper-truncated power law with scaling exponent, α, equal to 0.57, equivalent to a fractal dimension, D, equal to 1.71.
2. We provide a simple model for how heat from a hotspot, uniformly applied to the base of the lithosphere, produces a fractal distribution of seamount volumes on the seafloor. Events are initiated on a self-similar distribution of critical cells. We call this model Self-Similar Criticality (SSC).
3. Locations of magma migration through the lithosphere may be restricted to a self-similar distribution with a fractal dimension close to one. These locations may be associated with tectonic features that exhibit fractal properties.
4. The SSC model generates power law cumulative distributions with a range of scaling exponents and may be appropriate for many natural systems.

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